

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210 & TERM ID 2215)

ECT1026 – FIELD THEORY (RE, TE)

15 FEBRUARY 2023
2.30 p.m – 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 8 pages including cover page with **two** sections.
2. Attempt **ALL** questions in Section A.
3. Select **ONE** question from Section B.
4. The distribution of the marks for each question is given.
5. Please write all your answers in the answer booklet provided.

SECTION A
50 marks
(Answer ALL questions)

QUESTION 1

- a) Given a point $P(-2, 6, 3)$ and a vector $\mathbf{A} = y\hat{x} + (x + z)\hat{y}$
- i. Analyse P in cylindrical coordinates
[3 marks]
 - ii. Analyse P in spherical coordinates
[3 marks]
 - iii. Express $\mathbf{A}$ in cylindrical coordinates
[3 marks]
 - iv. Express $\mathbf{A}$ in spherical coordinates
[4 marks]
 - v. Evaluate $\mathbf{A}$ at P in Cartesian coordinates
[1 marks]
 - vi. Evaluate $\mathbf{A}$ at P in cylindrical coordinates
[2 marks]
 - vii. Evaluate $\mathbf{A}$ at P in spherical coordinate
[2 marks]
 - viii. Find the magnitude of $\mathbf{A}$
[1 marks]
- b) Find the gradient of $S = 4x^2e^{-z} + y^3$ and $M = R \cos \theta \sin \phi$ (identify the suitable coordinate system)
[3+3 marks]

Continued...

QUESTION 2

a) Sketch the hysteresis loop of a ferromagnetic materials and label the following points:

- i. Saturation
- ii. Retentivity
- iii. Coercivity

[3 marks]

b) Figure Q2 shows an iron core with relative permeability of 2000. The dimensions of the core are given in the figure. The width of the core is 7 cm. The lengths of the air gaps are 0.05 cm and 0.07 cm, respectively. The cross section of the air gaps is 5% bigger than the iron core. The coil has 300 turns and its current is 1.0 A.

- i. Draw the magnetic equivalent circuit

[2 marks]

- ii. Determine the magnetic flux in each column of the iron core.

[18 marks]

- iii. Determine the magnitude of flux density in the air gaps

[2 marks]

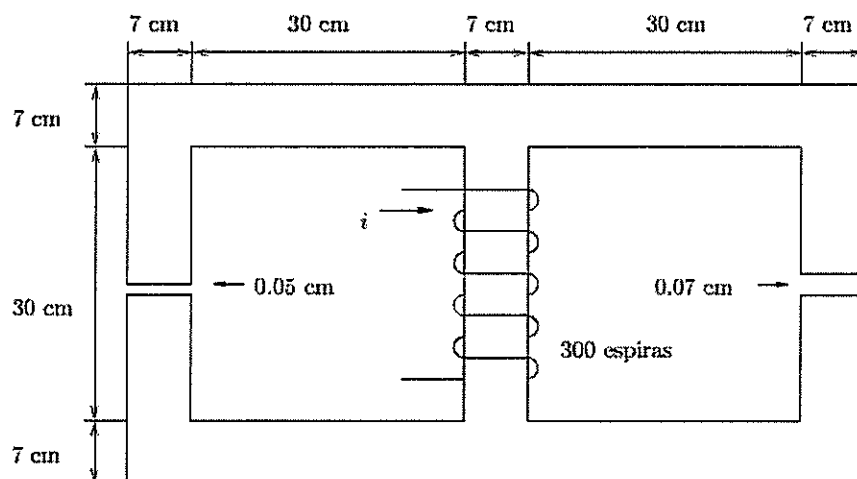


Figure Q2

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SECTION B
10 marks
(Choose and answer ONE question ONLY)

QUESTION 3

A cylindrical bar of silicon has a radius of 4mm and a length of 8cm. If a voltage of 5V is applied between the ends of the bar and $\mu_e = 0.13\text{m}^2/\text{Vs}$, $\mu_h = 0.05\text{m}^2/\text{Vs}$, $N_e = 1.5 \times 10^{16}\text{electron/m}^3$, and $N_h = N_e$, find the following:

- a) The conductivity of silicon. [2 marks]
- b) The current I flowing in the bar. [2 marks]
- c) The drift velocities $\mathbf{u}_e$, and $\mathbf{u}_h$. [2 marks]
- d) The resistance of the bar. [2 marks]
- e) The power dissipated in the bar. [2 marks]

Continued...

QUESTION 4

- a) A 100m long conductor of uniform cross-section has a voltage drop of 4V between its ends. If the density of the current flowing through it is $1.4 \times 10^6 \text{ A/m}^2$,

i. Find the conductivity.

[3 marks]

ii. Given Table Q4, identify the material of the conductor.

Table Q4

Material	Conductivity, σ (S/m)
<i>Conductors</i>	
Silver	6.2×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Iron	10^7
Mercury	10^6
Carbon	3×10^4
<i>Semiconductors</i>	
Pure germanium	2.2
Pure silicon	4.4×10^{-4}
<i>Insulators</i>	
Glass	10^{-12}
Paraffin	10^{-15}
Mica	10^{-15}
Fused quartz	10^{-17}

[1 marks]

- b) Describe and discuss the following:

- i. Conductor
- ii. Dielectric
- iii. Semiconductor

[2 + 2 + 2 marks]

Continued...

Appendix A : Physical constants, Vectors and Coordinate Transformations

Elementary charge	e	$1.60 \times 10^{-19} \text{ C}$
Permittivity constant	ϵ_0	$8.85 \times 10^{-12} \text{ F/m}$
Permeability constant	μ_0	$1.26 \times 10^{-6} \text{ H/m}$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}r d\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}R d\theta + \hat{\phi}R \sin \theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}dydz$ $d\vec{s}_y = \hat{y}dxdz$ $d\vec{s}_z = \hat{z}dxdy$	$d\vec{s}_r = \hat{r}rd\phi dz$ $d\vec{s}_\phi = \hat{\phi}drdz$ $d\vec{s}_z = \hat{z}rdrd\phi$	$d\vec{s}_R = \hat{R}R^2 \sin \theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta}R \sin \theta dR d\phi$ $d\vec{s}_\phi = \hat{\phi}R dR d\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2 \sin \theta dR d\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

Continued...

Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ , z)

$$\nabla V = \hat{r} \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \hat{r} \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ , ϕ)

$$\nabla V = \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times \vec{A} &= \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin \theta) A_\phi \end{vmatrix} \\ &= \hat{R} \frac{1}{R \sin \theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\theta} \frac{1}{R} \left[\frac{1}{\sin \theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R} (R A_\phi) \right] + \hat{\phi} \frac{1}{R} \left[\frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right] \end{aligned}$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

Continued...

Appendix C : Table of Integrals

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1$$

$$\int u dv = uv - \int v du$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax \sin ax \, dx = -\frac{1}{3a} \cos^3 ax$$

$$\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x$$

$$\int x \sqrt{x^2 \pm a^2} \, dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$$

$$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} \, dx = -\sqrt{a^2 - x^2}$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}}$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

End of Paper